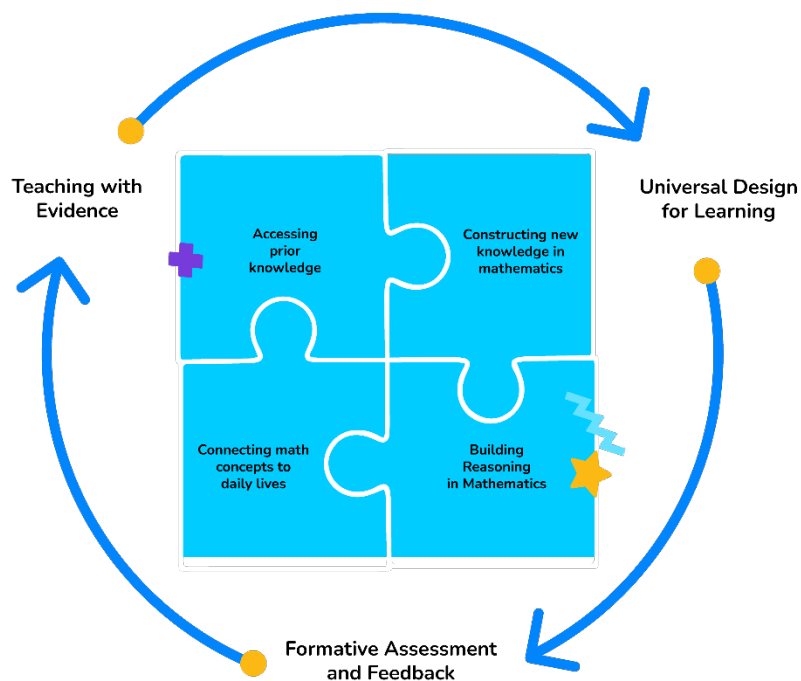




# Theory of Action References

DoodleMath is built upon a theory of action about how best to enhance the K-5 learning experience in mathematics. A **theory of action** is a collection of statements that hypothesize how intentional actions will result in change, based on insights from existing research. Collectively, our theory of action statements describe the philosophical foundation underpinning our approach to mathematics education.



**Goal:** Personalized learning supports can be costly and complicated for classroom teachers to implement. DoodleMath is a K–5 math practice solution that builds student fluency and confidence, personalized to each student’s learning needs in math. Geared at supplementing classroom instruction, DoodleMath provides a self-paced, fun, interactive, skill-based math program that adjusts to the unique learning needs of every student.

## Students

1. When students construct their knowledge of mathematics through the application of math practices, they deepen their understanding of mathematical ideas. [Evidence](#)
2. When students see how math concepts relate to their daily lives and personal interests, they are more motivated and engaged in learning. [Evidence](#)
3. When students engage in math activities and practices tailored to their learning needs, their self-confidence and interest in math increases as they simultaneously develop a mathematical growth mindset. [Evidence](#)
4. When students work through incrementally challenging math exercises, they build resilience and deeper understanding and have increased agency over their learning experience. [Evidence](#)

## Educators

5. When teachers provide opportunities for problem solving and application of math skills to real world math problems, educators construct learning experiences where self-efficacy and reflection are present and promote their students' acquisition of new math skills. [Evidence](#)
6. When teachers have access to effective formative assessment tools that enable them to identify students' knowledge gaps in math, they are better equipped to differentiate learning and use data to drive instructional decisions regularly in the classroom. [Evidence](#)

## Administrators

7. When school and district administrators invest in, prioritize, and support the use of personalized math programs, they support the development of a mathematical growth mindset for students and teachers. [Evidence](#)
8. When school and administrators have access to information about student progress in math learning and skill development, they promote and foster a culture of data-informed math instruction throughout the school/district. [Evidence](#)

## Parents

9. When parents have access to information about their student's progress in math, they become more in touch with their student's development in math and more confident in their ability to support their student. [Evidence](#)

# Theory of Action Statement 1

*When students construct their knowledge of mathematics through the application of math practices, they deepen their understanding of mathematical ideas.*

*The following studies and papers support this theory of action statement and provide guidance to our product development and product design, professional learning and partner success teams when creating, developing, and implementing our program.*

## 1. Mathematical Mindsets: Unleashing Students' Potential Through Creative Math, Inspiring Messages and Innovative Teaching

*Boaler, J. (2016). Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching. First edition. San Francisco, CA: Jossey-Bass.*

"Mathematics education expert and best-selling author Jo Boaler delivers a blueprint to banishing math anxiety and laying a foundation for mathematics success that anyone can build on. Perfect for students who have been convinced they are naturally "bad at math," the author offers a demonstration of how to turn self-doubt into self-confidence by relying on the "mindset" framework.

*Mathematical Mindsets* is based on thousands of hours of in-depth study and research into the most effective—and ineffective—ways to teach math to young people. This new edition also includes:

- Brand-new research from the last five years that sheds brighter light on how to turn a fear of math into an enthusiastic desire to learn.
- Developed ideas about ways to bring about equitable grouping in classrooms.
- New initiatives to bring 21st century mathematics to K-12 classrooms" – About this Book

## 2. Remediating Number combination and Word Problem Deficits Among Students with Mathematics Difficulties

*Fuchs, L.S., Powell, S.R., Seethaler, P.M., Cirino, P.T., Fletcher, J.M., Fuchs, D., Hamlett, C.L., & Zumeta, R.O. (2009). Remediating number combination and word problem deficits among students with mathematics difficulties: A randomized control trial. Journal of Educational Psychology, 101(3), 561-576.*

"The purposes of this study were to assess the efficacy of remedial tutoring for 3rd graders with mathematics difficulty, to investigate whether tutoring is differentially efficacious depending on students' math difficulty status (mathematics difficulty alone vs. mathematics plus reading difficulty), to explore transfer from number combination (NC) remediation, and to examine the transportability of the tutoring protocols. At 2 sites, 133 students were stratified on mathematics difficulty status and site and then randomly assigned to 3 conditions: control (no tutoring), tutoring on automatic retrieval of NCs (i.e., Math Flash), or tutoring on word problems with attention to the foundational skills of NCs, procedural calculations, and algebra (i.e., Pirate Math). Tutoring occurred for 16 weeks, 3 sessions per week and 20–30 min per session. Math Flash enhanced fluency with NCs with transfer to procedural computation but without transfer to algebra or word problems. Pirate Math enhanced word problem skill as well as fluency with NCs, procedural computation, and algebra. Tutoring was not differentially efficacious as a function of students' mathematics difficulty status. The tutoring protocols proved transportable across sites." – Abstract

### 3. A Case Study of Students' Creativity in Solving Mathematical Problems through Problem Based Learning

Maulidia, F., Johar, R. & Andariah, A. (2019). A case study of students' creativity in solving mathematical problems through problem-based learning. *Infinity Journal*

"Creativity could be interpreted as a person's cognitive abilities in solving problems by bringing up new ideas. The problems of students' math achievement lows are math presented as a finished product, ready to use, abstract and taught mechanistically. This case can be led to the creativity of the less developed students because students are not given the opportunity to think and use their ideas in solving mathematical problems. Problem Based Learning Model is a learning model that emphasizes the concept and information outlined from the academic discipline. The purpose of this study is to analyze students' creativity in solving mathematical problems through PBL...The data acquisition is categorized into 5 levels (highest level 4 and lowest level 0) which is analyzed descriptively. The results are three groups were at level 4 with very creative categories, one group is at level 3 with a creative category and another group is at level 2 with deeply creative enough category. To this conclusion is PBL model could cultivate the students' creativity in solving mathematical problems." – Abstract

### 4. Effects of a Tablet Game Intervention on Simple Addition and Subtraction Fluency in First Graders

van der Ven, F., Segers, E., Takashima, A., & Verhoeven, L. (2017). Effects of a tablet game intervention on simple addition and subtraction fluency in first graders. *Computers in Human Behavior*, 72, 200-207.

"In the present study, we aimed to playfully improve arithmetic fluency skills with a tablet game training. Participants were 103 grade 1 children from regular primary schools. The tablet game was tested with a pretest-posttest control group design and consisted of a racing game environment in which the player competed against a virtual opponent by rapidly solving addition and subtraction problems up to 20. During the 5-week intervention, one group ( $n = 52$ ) practiced with the game while another group ( $n = 51$ ) continued regular education without the game. Before, directly after, and three months after the intervention, we applied an arithmetic test to measure simple addition and subtraction skills in both symbolic (Arabic; 4) and non-symbolic (dots; :) number notations. The intervention group increased significantly more on *dot-subtraction* efficiency than the control group, an effect which was prominent directly after the intervention. Since i) dot-subtraction is considered to rely more on calculation than the other arithmetic types that we measured and ii) the dot problem-answer representations were not practiced during the intervention, our results suggest that the tablet game promoted arithmetic fluency by benefitting calculation efficiency rather than retrieval efficiency or the switch from calculation to retrieval". – Abstract

# Theory of Action Statement 2

*When students see how math concepts relate to their daily lives and personal interests, they are more motivated and engaged in learning.*

*The following studies and papers support this theory of action statement and provide guidance to our product development and product design, professional learning and partner success teams when creating, developing, and implementing our program.*

## 1. A Study of the Correlation Between STEM Career Knowledge, Mathematics Self-Efficacy, Career Interests, and Career Activities on the Likelihood of Pursuing a STEM Career Among Middle School Students.

*Blotnicky, K.A., Franz-Odendall, T., French, F., & Joy, P. (2018). A study of the correlation between STEM career knowledge, mathematics self-efficacy, career interests, and career activities on the likelihood of pursuing a STEM career among middle school students. International Journal of STEM Education, 5(22). doi: 10.1186/s40594-018-0118-3*

“A sample of 1448 students in grades 7 and 9 was drawn from public schools in Atlantic Canada to explore students’ knowledge of science and mathematics requirements for science, technology, engineering, and mathematics (STEM) careers. Also explored were their mathematics self-efficacy (MSE), their future career interests, their preferences for particular career activities, and their likelihood to pursue a STEM career...Analysis revealed that while older students had more knowledge about mathematics/science requirements for STEM careers, this knowledge was lacking overall. Also, students with higher MSE were more knowledgeable about STEM career requirements. Furthermore, students with higher MSE and STEM career knowledge were more likely to choose a STEM career. Students with greater interest in technical and scientific skills were also more likely to consider a STEM career than those who preferred career activities that involved practical, productive, and concrete activities...The results of this study show that students in middle school have a limited STEM career knowledge with respect to subject requirements and with respect to what sort of activities these careers involve. Furthermore, students with low MSE have a declining interest in STEM careers. Our data thus support the need to improve access to knowledge to facilitate students’ understanding of STEM careers and the nature of STEM work. Exposure of students to STEM careers can enhance their interest in pursuing careers involving science, technology, engineering, and mathematics.” – Abstract

## 2. An Examination of Middle School Students’ STEM Self-Efficacy with Relation to Interest and Perceptions of STEM

*Brown, P.L., Concannon, J.P., Marx, D., Donaldson, C.W., & Black, A. (2016). An examination of middle school students’ STEM self-efficacy with relation to interest and perceptions of STEM. Journal of STEM Education, 17(3), 27-38.*

“The purpose of this teacher research study is to ascertain students’ interest in STEM and beliefs about STEM before and after STEM specific instruction, explore possible differences in STEM self-efficacy by gender, and explore differences in STEM self-efficacy by group role. Our primary data sources include a modified attitudinal survey and modified perceptions of collaboration survey. We found differences in gender and students’ group roles to be related to self-efficacy, intentions to persist in STEM, perceptions of STEM and interests in STEM. Research on students’ self-efficacy and perceptions of group work has the potential to restructure how teachers design activities and teach students about collaboration.” – Abstract

### 3. Digital Games, Design, and Learning: A Systemic Review and Meta-Analysis

Clark, D. B., Tanner-Smith, E.E., & Killingsworth, S.S. (2016). *Digital games, design, and learning: A systemic review and meta-analysis*. *Review of Educational Research*, 86(1), 79-122.

"In this meta-analysis, we systematically reviewed research on digital games and learning for K–16 students. We synthesized comparisons of game versus nongame conditions (i.e., media comparisons) and comparisons of augmented games versus standard game designs (i.e., value-added comparisons). We used random-effects meta-regression models with robust variance estimates to summarize overall effects and explore potential moderator effects. Results from media comparisons indicated that digital games significantly enhanced student learning relative to nongame conditions. Results from value-added comparisons indicated significant learning benefits associated with augmented game designs, 95% confidence interval. Moderator analyses demonstrated that effects varied across various game mechanics characteristics, visual and narrative characteristics, and research quality characteristics. Taken together, the results highlight the affordances of games for learning as well as the key role of design beyond medium." – Abstract

### 4. Multicultural Children’s Literature as a Context for Teaching Mathematics

Leonard, J., Moore, C., Brooks, W. (2013, August 23). *Multicultural children’s literature as a context for teaching mathematics for cultural relevance in urban schools*. *Urban Review*, 46, 325-348. doi://10.1007/s11256-013-0264-3

"This article reports on a teacher-research study that used multicultural texts as a context for teaching mathematics for cultural relevance during an elementary mathematics methods course. The results of the study reveal that 28 % (5 out of 18) of the teacher candidates (TCs) chose books that were culturally contextual or culturally amenable. However, 89 % (16 out of 18) of the TCs chose texts that were mathematically robust or mathematically peripheral. Four focal TCs were selected to examine how they used the texts with children to teach mathematics concepts. Math lessons fostered academic success, cultural competence, and critical consciousness. Overall, the results of the study are mixed. We conclude that some TCs’ choice of texts may reflect indifference, passive resistance, low self-efficacy, school culture, and mixed messages from the teacher–researcher. Additional studies that include follow-up interviews and classroom observations are needed to determine the factors that influence TCs’ selections of multicultural texts and their implementation of culturally relevant pedagogy with students." – Abstract

### 5. Effects of Affiliation-, Achievement-, and Power-related Topics in Mathematical Word Problems on Students’ Performance, Task-related Values, and Expectancies

Scheidemann, B., Gasteiger, H., & Puca, R. (2022). *Effects of affiliation-, achievement-, and power-related topics in mathematical word problems on students’ performance, task-related values, and expectancies*. *PLoS ONE*, 17(6)

"A motivational downturn in mathematics during school has been well documented for many students. As a way to address this, creating personally relevant tasks is supposed to increase students’ motivation for mathematical tasks. According to recent research, topics relating to affiliation, achievement, and power are personally relevant for young people. Prior research showed that motive imagery in school tasks increases students’ task-related intrinsic value and success expectancies. The present study examined the effect of motive topics in word problems on students’ task performance. We contextualized mathematical tasks either with affiliation, achievement, and power topics or with neutral topics not related to motive topics. The tasks were comparable regarding their mean word count, text, and mathematical complexity. In three experimental studies, the students solved four tasks for each motive topic and neutral tasks, respectively. The dependent variables were task performance, intrinsic values, and expectancies of success. Repeated measures analyses of variance with the four-level, within-subjects factor motive imagery revealed positive effects of motive imagery in word problems on students’ task performance. This was particularly true for achievement-related tasks compared with neutral tasks. The results also indicated slightly positive effects for affiliation-related word problems on students’ performance. In addition, the intrinsic value for affiliation-related and achievement-related tasks was higher than for neutral tasks. Power imagery did not affect students’ task performance; it negatively affected students’ intrinsic value compared with neutral tasks. Task-related success expectancies were not influenced by motive imagery. The present study replicates and extends previous findings that indicate that tasks referring to affiliation and achievement imagery are more appealing to students and can benefit their performance." – Abstract

## 6. Multicultural Children’s Literature as a Context for Teaching Mathematics

*Uekawa, K., Borman, K., & Lee, R. (2007). Student engagement in U. S. urban high school mathematics and science classrooms: Findings on social organization, race and ethnicity. The Urban Review, 39(1), 1-43. DOI: 10.1007/s11256-006-0039-1*

“This paper reports results of intensive field work in urban high school mathematics and science classrooms based on research with students attending eight high schools located in large, disparate urban sites across the U.S. During the course of our observations and interviews we recorded students' activities as well as their impressions of classroom processes over the course of a week in each classroom using the Experience Sampling Method (ESM). ESM allows students to record information about their classroom experiences at intervals during daily mathematics and science classroom lessons. We measured levels of student engagement and examined relationships between student engagement and an array of predictors. We take a social organizational approach to interpreting classroom processes, examining the extent to which classroom activities influence student perception of class and communication among students and how these intervening factors affect student engagement during on-going classroom activities. Results suggest that there is variation between group members' reactions to classroom activities. Specifically, as an example, Latino Students in Chicago, Miami and El Paso were far more engaged and responsive to classroom lessons during the time they spent in small problem-solving groups during class. Student Engagement in High School Mathematics and Science.” – Abstract



# Theory of Action Statement 3

*When students engage in math activities and practices tailored to their learning needs, their self-confidence and interest in math increases as they simultaneously develop a mathematical growth mindset.*

*The following studies and papers support this theory of action statement and provide guidance to our product development and product design, professional learning and partner success teams when creating, developing, and implementing our program.*

## 1. Executive Functioning as a Predictor of Children’s Mathematics Ability: Inhibition, Switching and Working Memory

*Bull, R. & Scerif, G (2001). Executive functioning as a predictor of children’s mathematics ability: inhibition, switching and working memory. Developmental Neuropsychology, 19(3), 273-293.*

“Children’s mathematical skills were considered in relation to executive functions. Using multiple measures—including the Wisconsin Card Sorting Task (WCST), dual-task performance, Stroop task, and counting span—it was found that mathematical ability was significantly correlated with all measures of executive functioning, with the exception of dual-task performance. Furthermore, regression analyses revealed that each executive function measure predicted unique variance in mathematics ability. These results are discussed in terms of a central executive with diverse functions (Shallice & Burgess, 1996) and with recent evidence from Miyake, et al. (2000) showing the unity and diversity among executive functions. It is proposed that the particular difficulties for children of lower mathematical ability are lack of inhibition and poor working memory, which result in problems with switching and evaluation of new strategies for dealing with a particular task. The practical and theoretical implications of these results are discussed, along with suggestions for task changes and longitudinal studies that would clarify theoretical and developmental issues related to executive functioning.” – Abstract

## 2. Implicit Theories of Intelligence Predict Achievement Across an Adolescent Transition

*Blackwell, L.S., Trzesniewski, K.H., & Dweck, C.S. (2007). Implicit theories of intelligence predict achievement across an adolescent transition: A longitudinal study and an intervention. Child Development 78(1), 246-263.*

“Two studies explored the role of implicit theories of intelligence in adolescents’ mathematics achievement. In Study 1 with 373 7th graders, the belief that intelligence is malleable (incremental theory) predicted an upward trajectory in grades over the two years of junior high school, while a belief that intelligence is fixed (entity theory) predicted a flat trajectory. A mediational model including learning goals, positive beliefs about effort, and causal attributions and strategies was tested. In Study 2, an intervention teaching an incremental theory to 7th graders ( $N=48$ ) promoted positive change in classroom motivation, compared with a control group ( $N=43$ ). Simultaneously, students in the control group displayed a continuing downward trajectory in grades, while this decline was reversed for students in the experimental group.” - Abstract



### 3. Growth Mindset Tempers the Effects of Poverty on Academic Achievement

Claro, S., Paunesku, D., & Dweck, C.S. (2016). *Growth mindset tempers the effects of poverty on academic achievement. Proceedings of the National Academy of Sciences, 113(31), 8664-8668.*

“Two largely separate bodies of empirical research have shown that academic achievement is influenced by structural factors, such as socioeconomic background, and psychological factors, such as students’ beliefs about their abilities. In this research, we use a nationwide sample of high school students from Chile to investigate how these factors interact on a systemic level. Confirming prior research, we find that family income is a strong predictor of achievement. Extending prior research, we find that a growth mindset (the belief that intelligence is not fixed and can be developed) is a comparably strong predictor of achievement and that it exhibits a positive relationship with achievement across all of the socioeconomic strata in the country. Furthermore, we find that students from lower-income families were less likely to hold a growth mindset than their wealthier peers, but those who did hold a growth mindset were appreciably buffered against the deleterious effects of poverty on achievement: students in the lowest 10th percentile of family income who exhibited a growth mindset showed academic performance as high as that of fixed mindset students from the 80th income percentile. These results suggest that students’ mindsets may temper or exacerbate the effects of economic disadvantage on a systemic level.” - Abstract

### 4. Skills Underlying Mathematics: The Role of Executive Function in the Development of Mathematics Proficiency

Cragg, L., & Gilmore, C. (2014). *Skills underlying mathematics: The role of executive function in the development of mathematics proficiency. Trends in Neuroscience and Education, 3, 63-68.*

“The successful learning and performance of mathematics relies on a range of individual, social and educational factors. Recent research suggests that executive function skills, which include monitoring and manipulating information in mind (working memory), suppressing distracting information and unwanted responses (inhibition) and flexible thinking (shifting), play a critical role in the development of mathematics proficiency. This paper reviews the literature to assess concurrent relationships between mathematics and executive function skills, the role of executive function skills in the performance of mathematical calculations, and how executive function skills support the acquisition of new mathematics knowledge. In doing so, we highlight key theoretical issues within the field and identify future avenues for research.” – Abstract

### 5. Mindset: The new psychology of success

Dweck, C.S. (2006). *Mindset: The new psychology of success. New York, NY: Random House.*

“It has been shown that individuals with a growth mindset for learning see mistakes as opportunities to learn and improve, whereas for fixed-minded individuals mistakes indicate lack of ability. Earlier empirical research on mindsets includes both quantitative surveys and qualitative approaches with observations and stimulated recall method. During performance monitoring it is possible to probe event-related brain potentials (ERPs), enabling the investigation of the neural basis of students’ different reactions to mistakes. ERP studies have shown that growth mindset is associated with an enhancement of the error positivity (Pe), an ERP reflecting awareness of and attention allocation to mistakes. More growth-minded individuals also show superior accuracy after mistakes compared to those endorsing more of a fixed mindset. Most importantly, Pe amplitude mediates the relationship between mindset and post-error accuracy. These results suggest that neural activity indexing online awareness of and attention to mistakes is intimately involved in growth-minded individuals’ ability to rebound from mistakes. In this article we review and connect educational, psychological and neuroscientific points of view to investigate the role of mindsets related to learning.” – Abstract

## 6. Emotional and Motivational Relationship of Elementary Students to Mathematical Problem-Solving: A Person-Centered Approach

Hanin, V., & Nieuwenhoven, C. (2019). *Emotional and motivational relationship of elementary students to mathematical problem-solving: a person-centered approach*. *European Journal of Psychology of Education*, 34(4), 705-730.

“Recent literature has shown that achievement emotions, their regulation, and perceived competence play a compelling role in mathematics learning and achievement. Studies that have looked at these variables have, for the most part, adopted a person-centered approach, which examines relationships between variables found to a similar degree in all individuals of the group. Yet, scholars have outlined emotional inter-individual differences, in particular in terms of gender and past performance. The present study examined differences among upper elementary students in how achievement emotions are related to each other. Cluster analysis revealed four distinct profiles based on a sample of upper elementary students (N=354): those with high levels of positive emotions and low levels of negative emotions (*positive*); those with high levels of boredom and low levels of the other emotions (*bored*); those with high levels of nervousness, worry, and fear and low levels of positive emotions (*anxious*); and those with high levels of the six negative distinct emotions assessed and low levels of positive emotions (*resigned*). Analyses of variance showed that the first profile stood out advantageously from the last two regarding math performance and perceived competence. Findings regarding emotion regulation confirm the risky nature of the resigned profile. The bored profile ascribes no value, whether extrinsic or intrinsic to problem-solving tasks. Practical implications for educational practices and possible avenues for further research are discussed.” - Abstract

## 7. Individual Differences in Early Numeracy: The Role of Executive Functions and Subitizing

Kroesbergen, E., Van Luit, J., Van Lieshout, E., Van Loosbroek, E., & Van de Rijt, B. (2009). *Individual differences in early numeracy: The role of executive functions and subitizing*. *Journal of Psychoeducational Assessment*, 27(3), 226-236.

“In this study, the question was raised how basic cognitive processes are related to math abilities and how it can be best determined which children are at risk for developing those disabilities. The role of four distinct basic processes in the development of early mathematics was investigated: executive functions, fluid intelligence, subitizing, and language. The counting skills of 115 five- and six-year-old children were also assessed. The results showed that both executive functions and number sense were important factors in children's development of counting skills. Both executive functions and subitizing explained a significant part of variance in children's counting skills. IQ scores could not add further explanation to the variance in early math. The implications of this study are that it seems promising to use the concept of executive functions for the early identification of children at risk for math learning difficulties.” – Abstract

## 8. Emerging Themes from Early Research on Self-Efficacy Beliefs in School Mathematics

Michaelides, M. (2008). *Emerging themes from early research on self-efficacy beliefs in school mathematics*. *Electronic Journal of Research in Educational Psychology*, 6(1), 219-234.

“Self-efficacy, an individual's belief about his or her ability to engage in a task and accomplish it, has been investigated across multiple domains of human action. Students' perceived efficacy in school mathematics in particular, has been extensively explored by psychologists and educational researchers. We review literature from this field published prior to the year 2000 with the aim of extracting the fundamental themes that have drawn researchers' attention. The topics presented include the relationship between self-efficacy and mathematical performance, the interplay of self-efficacy with self-concept, properties of self-efficacy measurements, the relationship between self-efficacy and goal setting and modeling, the accuracy of self-efficacy judgments, and gender differences and career choice with respect to self-efficacy beliefs. We review the important concepts and notable findings for each theme and conclude with implications for educational practice and future research.” – Abstract

## 9. Role of Self-Efficacy and Self-Concept Beliefs in Mathematical Problem Solving: A Path Analysis

Pajares, F., & Miller, M. D. (1994). Role of self-efficacy and self-concept beliefs in mathematical problem solving: A path analysis. *Journal of Educational Psychology, 86*(2), 193-203.

“Path analysis was used to test the predictive and mediational role of self-efficacy beliefs in mathematical problem solving. Results revealed that math self-efficacy was more predictive of problem solving than was math self-concept, perceived usefulness of mathematics, prior experience with mathematics, or gender ( $N = 350$ ). Self-efficacy also mediated the effect of gender and prior experience on self-concept, perceived usefulness, and problem solving. Gender and prior experience influenced self-concept, perceived usefulness, and problem solving largely through the mediational role of self-efficacy. Men had higher performance, self-efficacy, and self-concept and lower anxiety, but these differences were due largely to the influence of self-efficacy, for gender had a direct effect only on self-efficacy and a prior experience variable. Results support the hypothesized role of self-efficacy in A. Bandura’s (1986) social cognitive theory.” – Abstract

## 10. Changes in the Relation Between Competence Beliefs and Achievement in Math Across Elementary School Years

Weidinger, A., Steinmayr, R., & Spinath, B. (2018). Changes in the Relation Between Competence Beliefs and Achievement in Math Across Elementary School Years. *Child Development, 89*(2), e138-e156.

“Math competence beliefs and achievement are important outcomes of school-based learning. Previous studies yielded inconsistent results on whether skill development, self-enhancement, or reciprocal effects account for the interplay among them. A development-related change in the direction of their relation in the early school years might explain the inconsistency. To test this, 542 German elementary school students ( $M=7.95$  years,  $SD=0.58$ ) were repeatedly investigated over 24 months from Grade 2 to Grade 4. Math competence beliefs declined and had a growing influence on subsequent math grades. This suggests changes in the dominant direction of the relation from a skill development of a reciprocal effects model during elementary school. Findings are discussed with regard to their theoretical and practical implications.” – Abstract

## 11. Mind in Society: The Development of Higher Psychological Processes.

Vygotsky, L. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.

“The great Russian psychologist **L. S. Vygotsky** has long been recognized as a pioneer in developmental psychology. But somewhat ironically, his theory of development has never been well understood in the West. *Mind in Society* should correct much of this misunderstanding. Carefully edited by a group of outstanding Vygotsky scholars, the book presents a unique selection of Vygotsky’s important essays, most of which have previously been unavailable in English. The Vygotsky who emerges from these pages can no longer be glibly included among the neobehaviorists. In these essays he outlines a dialectical-materialist theory of cognitive development that anticipates much recent work in American social science. The mind, Vygotsky argues, cannot be understood in isolation from the surrounding society. Man is the only animal who uses tools to alter his own inner world as well as the world around him. From the handkerchief knotted as a simple mnemonic device to the complexities of symbolic language, society provides the individual with technology that can be used to shape the private processes of mind. In *Mind in Society* Vygotsky applies this theoretical framework to the development of perception, attention, memory, language, and play, and he examines its implications for education. The result is a remarkably interesting book that is bound to renew Vygotsky’s relevance to modern psychological thought.” – About this Book

# Theory of Action Statement 4

*When students work through incrementally challenging math exercises, they build resilience and deeper understanding and have increased agency over their learning experience.*

*The following studies and papers support this theory of action statement and provide guidance to our product development and product design, professional learning and partner success teams when creating, developing, and implementing our program.*

## 1. Dyscalculia: From Brain to Education

*Butterworth, B., Varma S., & Laurillard D. (2011). Dyscalculia: From brain to education. Science, 332, 1049-1053.*

“Recent research in cognitive and developmental neuroscience is providing a new approach to the understanding of dyscalculia that emphasizes a core deficit in understanding sets and their numerosities, which is fundamental to all aspects of elementary school mathematics. The neural bases of numerosity processing have been investigated in structural and functional neuroimaging studies of adults and children, and neural markers of its impairment in dyscalculia have been identified. New interventions to strengthen numerosity processing, including adaptive software, promise effective evidence-based education for dyscalculic learners.” - Abstract

## 2. School Readiness and Later Achievement

*Duncan, G.J., Dowsett, C.J., Classens, A., Magnuson, K., Huston, A.C., Klebanov, P., Pagina, L.S., Feinstein, L., Engel, M., Brooks-Gunn, J., Sexton, H., Duckworth, K., & Japel, C. (2007). School readiness and later achievement. Developmental Psychology, 43(6), 1428-1446.*

“Using 6 longitudinal data sets, the authors estimate links between three key elements of school readiness--school-entry academic, attention, and socioemotional skills--and later school reading and math achievement. In an effort to isolate the effects of these school-entry skills, the authors ensured that most of their regression models control for cognitive, attention, and socioemotional skills measured prior to school entry, as well as a host of family background measures. Across all 6 studies, the strongest predictors of later achievement are school-entry math, reading, and attention skills. A meta-analysis of the results shows that early math skills have the greatest predictive power, followed by reading and then attention skills. By contrast, measures of socioemotional behaviors, including internalizing and externalizing problems and social skills, were generally insignificant predictors of later academic performance, even among children with relatively high levels of problem behavior. Patterns of association were similar for boys and girls and for children from high and low socioeconomic backgrounds.” - Abstract

## 3. Challenging Conceptual Understanding in a Complex System: Supporting Young Students to Address Extended Mathematical Inquiry Problems

*Fielding, J., & Makar, K. (2022). Challenging conceptual understanding in a complex system: supporting young students to address extended mathematical inquiry problems. Instructional Science, 50(1), 35-61.*

“Conceptual challenge is often considered a necessary ingredient for promoting deep learning in an inquiry-based environment. However, challenge alone does not support conceptual development. In this paper, we draw on complexity theory as a theoretical lens to explore how a primary teacher facilitated students’ conceptual change through repeated cycles of challenge and support to develop increasingly robust concepts. Data are drawn from a primary class as they were developing initial understandings of distribution, informal statistical inference and sampling variability in the process of solving an extended mathematical inquiry problem. Data included classroom video, researcher journal and student work samples. The findings suggest two benefits to guiding students through multiple iterations of challenge and guidance; the opportunity to provoke and guide richer mathematical concepts; and the opportunity to provide earlier exposure to advanced mathematical concepts. Building on this research, we argue for the value of multiple iterations of challenge-support phases to develop increasingly robust understanding over time.” – Abstract

## 4. Contributions of learning to human development

Gagne, R.M. (1968). *Contributions of learning to human development. Psychological Review*, 5(3), 177-191.

“Critically considers models of human intellectual development. 2 commonly contrasted models are growth readiness, associated with the writings of Hall and Gesell, and cognitive adaptation, developed by Piaget. A 3rd model, cumulative learning, is discussed. Applications of this model including an analysis of the conservation of liquid volume as a learned capability of young children are noted. Implications of the cumulative learning model for transfer of learning are described.” – Abstract

## 5. Early Math Matters: Kindergarten Number Competence and Later Mathematics Outcomes

*Early math matters: Kindergarten number competence and later mathematics outcomes*  
Jordan, N.C., Kaplan, D., Ramineni, C., & Locuniak, M.N. (2009). *Early math matters: Kindergarten number competence and later mathematics outcomes. Developmental Psychology*, 45(3), 850-867.

“Children’s number competencies over 6 time points, from the beginning of kindergarten to the middle of 1st grade, were examined in relation to their mathematics achievement over 5 later time points, from the end of 1st grade to the end of 3rd grade. The relation between early number competence and mathematics achievement was strong and significant throughout the study period. A sequential process growth curve model showed that kindergarten number competence predicted rate of growth in mathematics achievement between 1st and 3rd grades as well as achievement level through 3rd grade. Further, rate of growth in early number competence predicted mathematics performance level in 3rd grade. Although low-income children performed more poorly than their middle-income counterparts in mathematics achievement and progressed at a slower rate, their performance and growth were mediated through relatively weak kindergarten number competence. Similarly, the better performance and faster growth of children who entered kindergarten at an older age were explained by kindergarten number competence. The findings show the importance of early number competence for setting children’s learning trajectories in elementary school mathematics.” – Abstract

## 6. Effectiveness of Cognitive Tutor Algebra I at Scale

Pane, J.F., Griffin, B.A., McCaffrey, D.F., & Karam, R. (2014). *Effectiveness of cognitive tutor Algebra I at scale. Educational Evaluation and Policy Analysis*, 36(2), 127-144.

“This article examines the effectiveness of a technology-based algebra curriculum in a wide variety of middle schools and high schools in seven states. Participating schools were matched into similar pairs and randomly assigned to either continue with the current algebra curriculum for 2 years or to adopt Cognitive Tutor Algebra I (CTAI), which uses a personalized, mastery-learning, blended-learning approach. Schools assigned to implement CTAI did so under conditions similar to schools that independently adopt it. Analysis of posttest outcomes on an algebra proficiency exam finds no effects in the first year of implementation but finds evidence in support of positive effects in the second year. The estimated effect is statistically significant for high schools but not for middle schools; in both cases, the magnitude is sufficient to improve the median student’s performance by approximately eight percentile points.” – Abstract

## 7. Maximizing Competency and Blended Learning

Patrick, S., & Sturgis, C. (2015). *Maximizing competency and blended learning: Insights from experts*. Vienna, VA: International Association of K-12 Online Learning

“Students will face enormous challenges in the coming years—from an economy shaped by ever-advancing technologies to the impact of globalization—and need the strongest foundation of academic, technical, and problem-solving skills we can offer. In an effort to improve their educational experiences, schools across the country are exploring and implementing new approaches, many of which share a common goal: to provide greater personalization and ensure that each and every student has the knowledge, skills, and competencies to succeed. Personalized learning, blended learning, and competency-based learning are becoming of increasing interest to district leaders at the front of transformation efforts. These efforts rely on support and direction from a limited pool of technical assistance providers in the field. Technical assistance providers are individuals or organizations with expertise in their respective fields who are charged with providing implementation assistance such as strategic planning, training, resources, and direct assistance to schools and districts. Each provider has expertise in some areas; few have expertise in all of them. Thus, the implication for districts is that the transformation process is staggered to allow for the implementation of one major strategy and then another, rather than taking on a broad-sweeping comprehensive approach. In May 2014, CompetencyWorks brought together twenty-three technical assistance providers to examine their catalytic role in implementing next generation learning models, share each other's knowledge and expertise about blended learning and competency education, and discuss next steps to move the field forward with a focus on equity and quality. The strategy maintains that by building the knowledge and networks of technical assistance providers, these groups can play an even more catalytic role in advancing the field. The objective of the convening was to help educate and level set the understanding of competency education and its design elements, as well as to build knowledge about using blended learning modalities within competency-based environments. This paper attempts to draw together the wide-ranging conversations from the convening to provide background knowledge for educators to understand what it will take to transform from traditional to personalized, competency-based systems that take full advantage of blended learning. The primary focus is to address the key considerations that face districts as they move forward. The authors consider the discussion offered here as a first step in a very steep learning curve that they will be making to fully maximize competency-based structures and blended learning modalities.” - Abstract

## 8. How Numeracy Influences Risk Comprehension and Medical Decision Making

Reyna, F., Nelson, W.L., Han P.K., & Dieckmann, N.F. (2009). *How numeracy influences risk comprehension and medical decision making*. *Psychological Bulletin*, 135(6), 943-973.

“We review the growing literature on health numeracy, the ability to understand and use numerical information, and its relation to cognition, health behaviors, and medical outcomes. Despite the surfeit of health information from commercial and noncommercial sources, national and international surveys show that many people lack basic numerical skills that are essential to maintain their health and make informed medical decisions. Low numeracy distorts perceptions of risks and benefits of screening, reduces medication compliance, impedes access to treatments, impairs risk communication (limiting prevention efforts among the most vulnerable), and, based on the scant research conducted on outcomes, appears to adversely affect medical outcomes. Low numeracy is also associated with greater susceptibility to extraneous factors (i.e., factors that do not change the objective numerical information). That is, low numeracy increases susceptibility to effects of mood or how information is presented (e.g., as frequencies vs. percentages) and to biases in judgment and decision making (e.g., framing and ratio bias effects). Much of this research is not grounded in empirically supported theories of numeracy or mathematical cognition, which are crucial for designing evidence-based policies and interventions that are effective in reducing risk and improving medical decision making. To address this gap, we outline four theoretical approaches (psychophysical, computational, standard dual-process, and fuzzy trace theory), review their implications for numeracy, and point to avenues for future research.” – Abstract



# Theory of Action Statement 5

*When teachers provide opportunities for problem solving and application of math skills to real world math problems, educators construct learning experiences where self-efficacy and reflection are present and promoted in students' acquisition of new math skills.*

*The following studies and papers support this theory of action statement and provide guidance to our product development and product design, professional learning and partner success teams when creating, developing, and implementing our program.*

## 1. Why is Teaching with Problem Solving Important to Student Learning?

Cai, J. & Lester, F. (2010). *Why is teaching with problem solving important to student learning?* National Council of Teachers of Mathematics. [URL](#)

“Developing students' abilities to solve problems is not only a fundamental part of mathematics learning across content areas but also an integral part of mathematics learning across grade levels. Beginning in preschool or kindergarten, students should be taught mathematics in a way that fosters understanding of mathematics concepts and procedures and solving problems. In fact, there is strong evidence that even very young students are quite capable of exploring problem situations and inventing strategies to solve the problems (e.g., Ben-Chaim et al., 1998; Cai, 2000; Carpenter et al., 1998; Kamii & Housman, 1989; Maher & Martino, 1996; Resnick, 1989) ...

Research clearly suggests that problem solving should not be taught as a separate topic in the mathematics curriculum. In fact, research tells us that teaching students to use general problem-solving strategies has little effect on their success as problem solvers. Thus, problem solving must be taught as an integral part of mathematics learning, and it requires a significant commitment in the curriculum at every grade level and in every mathematical topic. In addition to making a commitment to problem solving in the mathematics curriculum, teachers need to be strategic in selecting appropriate tasks and orchestrating classroom discourse to maximize learning opportunities. In particular, teachers should engage students in a variety of problem-solving activities: (a) finding multiple solution strategies for a given problem, (b) engaging in mathematical exploration, (c) giving reasons for their solutions, and (d) making generalizations. Focusing on problem solving in the classroom not only impacts the development of students' higher-order thinking skills but also reinforces positive attitudes. Finally, there is no evidence that we should worry that students sacrifice their basic skills if teachers focus on developing their mathematical problem-solving skills.” - Conclusion

## 2. Crosscutting Concepts and Achievement: Is a sense of Size and Scale Related to Achievement in Science and Mathematics

Chesnutt, K., Gail Jones, M., Corin, E., Hite, R., Childers, G., Perez, M., Cayton, E., & Ennes, M. (2019). *Crosscutting concepts and achievement: Are a sense of size and scale related to achievement in science and mathematics?* *Journal of Research in Science Teaching*, 56(3), 302-321.

“This study exemplified the relationship between students' (N=229) concepts of size and scale and students' achievement in science and mathematics over a 3-year period. Size and scale are considered one of the big ideas in science that permeates disparate science and mathematics content areas, yet little is known about the relationship between students' conceptualization of size and scale and students' achievement in science and mathematics. The study used a modified panel longitudinal design to follow the same class of students over a 3-year period. The goal was to explore whether understandings of size and scale are related to achievement in mathematics and science. Results indicated a strong positive significant relationship existed between students' understanding of size and scale and students' science achievement in grades 5 and 8. There was a positive significant relationship between students' concepts of size and scale and students' mathematics achievement in grades 5, 6, 7, and 8. An examination of the relationships is included as well as a discussion of the integration of crosscutting concepts into science and mathematics instruction as a way to support deep learning.” – Abstract



### 3. The Role of Motion-Based Metaphors in Enhancing Mathematical Thought: A Perspective from Embodiment Theories of Cognition

*Khatin-Zadeh, O., Marmolejo-Ramos, F., & Trenholm, S. (2022). The Role of Motion-Based Metaphors in Enhancing Mathematical Thought: A Perspective from Embodiment Theories of Cognition. Journal of Cognitive Enhancement, 6(4), 455-462.*

“Motion-based metaphors help explain a single, often static, concept (e.g., number, mathematical function, limit of function, continuity of function) in terms of a human motion. In mathematics, many mathematical concepts, such as function and continuity, are described in terms of graphical representations. Although these graphical representations are static, they can be transformed into motion events and understood as motions by motion-based metaphors. This can be done by either using a hand gesture to depict the graphical representation of the target concept or by mentally simulating hand movements that depict the graphical representation. By employing these mechanisms, the motor system becomes engaged in the process of aiding the learner to understand static mathematical concepts (concepts that are defined in terms of non-moving mathematical objects), acting as a cognitive resource to ground and understand non-motion mathematical concepts. In this paper, we theorize that visual representations of mathematical concepts have varying degrees of what we term to be “motor strength” whereby, for example, curves of functions may be either strongly or weakly motoric depending on the degree to which they aid in the development of associated deep mathematical thinking.” – Abstract

### 4. Improving Mathematical Problem Solving in Grades 4 through 8: A Practice Guide

*Woodward, J., Beckmann, S., Driscoll, M., Franke, M., Herzig, P., Jitendra, A., Koedinger, K. R., & Ogbuehi, P. (2012). Improving mathematical problem solving in grades 4 through 8: A practice guide (NCEE 2012-4055). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education. Retrieved from [http://ies.ed.gov/ncee/wwc/publications\\_reviews.aspx#pubsearch/](http://ies.ed.gov/ncee/wwc/publications_reviews.aspx#pubsearch/)*

This practice guide from the Institute of Education Sciences What Works Clearinghouse provides five evidence-based recommendations for improving students' mathematical problem solving in grades 4 through 8. This guide is geared toward teachers, math coaches, other educators, and curriculum developers who want to improve the mathematical problem solving of students. 1) Prepare problems and use them in whole-class instruction; 2) Assist students in monitoring and reflecting on problem-solving; 3) Teach students how to use visual representations; 4) Expose students to multiple problem-solving strategies; and 5) Help students recognize and articulate mathematical concepts and notation.

# Theory of Action Statement 6

*When teachers have access to effective formative assessment tools that enable them to identify students' knowledge gaps in math, they are better equipped to differentiate learning and use data to drive instructional decisions regularly in the classroom.*

*The following studies and papers support this theory of action statement and provide guidance to our product development and product design, professional learning and partner success teams when creating, developing, and implementing our program.*

## 1. Fundamentals of Formative Assessment for Classroom Teachers

*Bartz, D.E. (2017). Fundamentals of formative assessment for classroom teachers. National Forum of Teacher Education Journal, 27(3), 1-10. URL*

"Formative assessment is a planned process in which assessment results are used to measure students' progress toward mastering learning targets and for teachers to adjust instruction and content. The formative assessment process is composed of the following nine steps: (1) determining the content to be learned and taught, (2) identifying and clearly describing assessment criteria for the content, (3) determining instructional strategies as a reference point to commence instruction, (4) sharing with students their role in formative assessment, (5) administering a pre- assessment/pretest (if applicable), (6) implementing the instructional strategies, (7) collecting formative assessment data, (8) providing students with feedback from the formative assessment pointed toward learning targets, and (9) readjusting instruction for students based on the formative assessment feedback. Supplementing formative assessment through using technology can make it even more effective and has the potential to lighten the teacher's workload. A close and objective analysis of the state- of-the-art benchmark or interim assessments reveals that they can be useful for improving student learning." -Abstract

## 2. Differentiated Instruction in a Data-based Decision-making Context

*Faber, J. M., Glas, C.A., & Visscher, A.J. (2018). Differentiated instruction in a data-based decision-making context. School Effectiveness and School Improvement, 29(1), 43-63. URL*

"In this study, the relationship between differentiated instruction, as an element of data-based decision making, and student achievement was examined. Classroom observations (n = 144) were used to measure teachers' differentiated instruction practices and to predict the mathematical achievement of 2nd- and 5th-grade students (n = 953). The analysis of classroom observation data was based on a combination of generalizability theory and item response theory, and student achievement effects were determined by means of multilevel analysis. No significant positive effects were found for differentiated instruction practices. Furthermore, findings showed that students in low-ability groups profited less from differentiated instruction than students in average or high-ability groups. Nevertheless, the findings, data collection, and data-analysis procedures of this study contribute to the study of classroom observation and the measurement of differentiated instruction." -Abstract

## 3. Formative Assessment: Making it Happen in the Classroom

*Heritage, M. (2021). Formative assessment: Making it happen in the classroom. Corwin Press.*

"Formative assessment is a process used by teachers and students to keep learning moving forward. In the 10 years since the first edition of Formative Assessment was published, the practice has become a mainstay in classrooms, but that does not mean that it is easy. Education expert Margaret Heritage walks readers through every step of implementation and offers numerous examples that illustrate formative assessment practices across a range of subjects and grade levels. She explains how to articulate learning progressions, goals, and success criteria; select assessment strategies and provide quality feedback; engage students in self-assessment and self-management; and create an environment that values feedback as part of the learning process." -Abstract

#### 4. The Power of Assessment for Learning: Twenty Years of Research and Practice in UK and US Classrooms

Heritage, M., & Harrison, C. (2019). *The power of assessment for learning: Twenty years of research and practice in UK and US classrooms.*

“Twenty years after the publication of “Inside the Black Box,” the landmark review of formative classroom assessment, international education experts Christine Harrison and Margaret Heritage tackle assessment for learning (AfL) anew, with fresh insights gained from two decades of research, theory, and classroom practice. “The Power of Assessment for Learning: Twenty Years of Research and Practice in UK & US Classrooms” examines the practices and processes of formative assessment over time in both countries, evaluates the benefits accrued to teaching and learning, and considers future developments in growing and sustaining AfL practice. It features: (1) Key AfL ideas, approaches, and supports; (2) Vignettes of classroom practice that illustrate AfL in action in the U.K. and U.S.; and (3) Practice-based evidence to enrich understanding of AfL from both the teacher’s and the student’s perspective. Focused on student-centeredness and rich with classroom examples, this book is a ‘sounding board’ for educators to explore and reflect on their own AfL practices and beliefs.” - Abstract

#### 5. From Evidence to Action: A Seamless Process in Formative Assessment?

Heritage, M., Kim, J., Vendlinski, T., & Herman, J. (2008). *From evidence to action: A seamless process in formative assessment? (CRESST report 741). Educational Measurement: Issues and Practice, 28(3), 24-31. URL*

“Based on the results of a generalizability study (G study) of measures of teacher knowledge for teaching mathematics developed at The National Center for Research, on Evaluation, Standards, and Student Testing (CRESST) at the University of California, Los Angeles, this report provides evidence that teachers are better at drawing reasonable inferences about student levels of understanding from assessment information than they are in deciding the next instructional steps. We discuss the implications of the results for effective formative assessment and end with considerations of how teachers can be supported to know what to teach next.” - Abstract

#### 6. Effective Formative Assessment for Elementary Grade Classrooms

Klute, M. M., & Apthorp, H. S. (2018). *Effective Formative Assessment for Elementary Grade Classrooms: A Research Review. AERA Online Paper Repository. URL*

“This paper discusses practical implications of formative assessment effectiveness research in grades one through six. Formative assessment effectiveness is a timely topic for state and district leaders as they develop implementation plans for Every Student Succeeds Act (ESSA), including their approaches to supporting school-based practitioners (Shepard, Penuel, & Davidson, 2017). This paper addresses one overarching research question: What effectiveness claims can we responsibly make about formative assessment? To improve on prior research, we adopted Bennett’s (2011) recommendations for conducting research on the effectiveness of formative assessment, including: precisely defining formative assessment, considering whether studies of the impact of formative assessment were conducted with sufficient quality to support causal inferences, and describing concrete instantiations of effective formative assessment.” - Abstract

## 7. Formative Assessment and Elementary School Student Academic Achievement: A Review of the Evidence

Klute, M., Apthorp, H., Harlacher, J., & Reale, M. (2017). *Formative Assessment and Elementary School Student Academic Achievement: A Review of the Evidence (REL 2017-259)*. Regional Educational Laboratory Central. URL

"[This] review identifies studies that examine the effectiveness of formative assessment and provides an overall average estimate of its effectiveness. This review also compares the effectiveness of different types of formative assessment... The review team identified 23 studies that it determined had been conducted rigorously enough to have confidence that the formative assessment interventions caused the observed effects on student outcomes. Twenty-two of the studies compared academic outcomes for students participating in formative assessment with academic outcomes for students who did not participate in formative assessment. Nineteen of the 22 studies provided enough information to calculate an effect size, which describes the magnitude of the effect of the intervention. When examining the results across these 19 studies, the review team concluded that: (1) Overall, formative assessment had a positive effect on student academic achievement. On average across all the studies, students who participated in formative assessment performed better on measures of academic achievement than those who did not; (2) Formative assessment used during math instruction had larger effects, on average, than did formative assessment used during reading and writing instruction; (3) Across all subject areas (math, reading, and writing), formative assessment had larger effects on student academic achievement when other agents, such as a teacher or a computer program, directed the formative assessment; (4) For math, both student-directed formative assessment and formative assessment directed by other agents were effective; (5) For reading, other-directed formative assessment was more effective than student-directed formative assessment; and (6) For writing, the effect of other-directed formative assessment on student academic achievement was small, and not enough evidence was available to determine the effectiveness of student-directed formative assessment." - Abstract

## 8. Differentiated Instruction in Secondary Education: A Systematic Review of Research Evidence

Smale-Jacobse, A. E., Meijer, A., Helms-Lorenz, M., & Maulana, R. (2019). *Differentiated instruction in secondary education: A systematic review of research evidence*. *Frontiers in psychology*, 10, 2366. URL

"Differentiated instruction is a pedagogical-didactical approach that provides teachers with a starting point for meeting students' diverse learning needs. Although differentiated instruction has gained a lot of attention in practice and research, not much is known about the status of the empirical evidence and its benefits for enhancing student achievement in secondary education. The current review sets out to provide an overview of the theoretical conceptualizations of differentiated instruction as well as prior findings on its effectiveness. Then, by means of a systematic review of the literature from 2006 to 2016, empirical evidence on the effects of within-class differentiated instruction for secondary school students' academic achievement is evaluated and summarized. After a rigorous search and selection process, only 14 papers about 12 unique empirical studies on the topic were selected for review. A narrative description of the selected papers shows that differentiated instruction has been operationalized in many different ways. The selection includes studies on generic teacher trainings for differentiated instruction, ability grouping and tiering, individualization, mastery learning, heterogeneous grouping, and remediation in flipped classroom lessons. The majority of the studies show small to moderate positive effects of differentiated instruction on student achievement...These empirical findings give some indication of the possible benefits of differentiated instruction. However, they also point out that there are still severe knowledge gaps. More research is needed before drawing convincing conclusions regarding the effectiveness and value of different approaches to differentiated instruction for secondary school classes." – Abstract.

## 9. Differentiating Instruction in a Mathematics Classroom: Its Effects on Senior High School Learners' Academic Performance and Engagement in Basic Calculus

Tambaoan, R. S., & Gaylo, D. N. (2019). *Differentiating instruction in a mathematics classroom: Its effects on senior high school learners' academic performance and engagement in basic calculus*. *International journal of English and education*, 8(2), 272-286. URL

"A quasi-experimental study was conducted to investigate the effects of differentiated instruction, a strategy that may cater to learners' diversity, towards their academic performance and engagement in Basic Calculus. It was participated by sixty Grade 11 learners in the Science, Technology, Engineering and Mathematics (STEM) strand of Bukidnon State University Secondary School, Malaybalay City during the second semester of the school year 2017-2018. Lessons on differentiation and its applications were developed. Researcher-made academic performance test and engagement scale were evaluated by a panel of experts and underwent validity and reliability analysis. The gathered data were analyzed and interpreted using appropriate statistical techniques: mean, standard deviation, frequency, percentage, one-way analysis of covariance (ANCOVA) and paired t-test. The results revealed that the learners' academic performance when taught using differentiated instruction was Very Satisfactory; while learners taught with the conventional instruction was Fairly Satisfactory. There was a statistically significant difference in the academic performance between the two groups of learners, in favor of those taught with differentiated instruction. Moreover, the engagement level of the learners in the experimental group was Moderate before and after the intervention, and there was a statistically significant difference between them attributed to differentiated instruction." – Abstract

## 10. Assessment in a Differentiated Classroom

Tomlinson, C. A., & Moon, T. (2014). *Assessment in a differentiated classroom. Proven programs in education: Classroom management and assessment*, 1-5. URL

"It's generally accepted that one mission of schooling is to help learners develop competence and confidence with important knowledge, understanding, and skills designed to help them relate more meaningfully to the world they live in and prepare them to be good stewards of that world. Fundamental to that mission is ensuring that educators have a sound understanding of the roles of curriculum design, assessment, and instructional planning in student success. This paper highlights attributes of quality classroom practice within and among these three areas. It pays particular attention to critical intersections between formative assessment and instructional planning for teachers who seek to support the success of a broad range of learners in today's culturally and academically diverse classrooms." -Introduction

## 11. The Bridge Between Today's Lesson and Tomorrow's

Tomlinson, C. A. (2016). *The bridge between today's lesson and tomorrows. On Formative Assessment: Readings from messages, ional Leadership (EL Essentials)*, 14.

"A lot of people talk about the value of formative assessment, but Carol Ann Tomlinson points out that, too often, it is reduced to a mechanism for raising end-of-year-test scores when it should be an ongoing exchange between a teacher and his or her students designed to help students grow. When aligned with current content goals, it can help teachers and students make near-term adjustments so that student learning can proceed. Tomlinson offers 10 principles to help teachers apply sound formative assessment practices: (1) Help students understand the role of formative assessment; (2) Begin with clarity about what students need to know, understand, and be able to do; (3) Make room for student differences; (4) Provide instructive feedback to students; (5) Make feedback user-friendly; (6) Assess persistently; (7) Engage students with formative assessment; (8) Look for patterns; (9) Plan instruction around content requirements and student needs; and (10) Repeat the process." - Abstract

# Theory of Action Statement 7

*When school and district administrators invest in, prioritize, and support the use of personalized math programs, they support the development of a mathematical growth mindset for students and teachers.*

*The following studies and papers support this theory of action statement and provide guidance to our product development and product design, professional learning and partner success teams when creating, developing, and implementing our program.*

## 1. Efficacy of an Adaptive Game-Based Math Learning App to Support Personalized Learning and Improve Early Elementary School Students' Learning

*Bang, H., Li, L., & Flynn, K. (2022). Efficacy of an Adaptive Game-Based Math Learning App to Support Personalized Learning and Improve Early Elementary School Students' Learning. Early Childhood Education Journal, OnlineFirst, 1-16.*

"This study investigates whether using My Math Academy, which provides personalized content and adaptive embedded assessments to support existing curricula, can improve learning outcomes and engagement for kindergarten and first grade students (N=505 treatment, 481 control). Findings indicate that students using My math Academy made significant learning gains in math relative to children who did not. More skills mastered in My math Academy was associated with greater learning gains on the external assessment, with the greatest impacts among students with lower levels of math knowledge, where there was more room for growth and on the most difficult skills. Teachers surveyed found My Math Academy easy to use in their classrooms and recognized it as a valuable learning resource that supplemented their existing curricula to improve students' engagement, motivation, and confidence in learning math." – Abstract

## 2. School Instructional Climate and Student Achievement: An Examination of Group Norms for Differentiated Instruction

*Goddard, Y., Goddard, R., & Kim, M. (2015). School instructional climate and student achievement: An examination of group norms for differentiated instruction. American Journal of Education, 122(1), 111-131.*

"The purpose of this article is to investigate the relationship between school instructional climate and students' fifth-grade mathematics and reading achievement, with a particular emphasis on norms for practice consistent with differentiated instruction. Using data from a stratified random sample of Michigan elementary schools, we employed multilevel structural equation modeling to test our hypothesis. Our results confirmed that school norms for teaching practices consistent with differentiating instruction were positively and significantly associated with differences among schools in mathematics and reading achievement. The results are important for educators seeking knowledge about school characteristics and instructional practices associated with improved student achievement." – Abstract

## 3. Engaging Primary School Students in Mathematics: Can iPads Make a Difference?

*Hilton, A. (2018). Engaging primary school students in mathematics: Can iPads make a difference? International Journal of Science and Mathematics Education, 16, 145-165.*

"Research on the impact of the integration of technologies such as iPads on primary students' attitudes and engagement in mathematics is limited. Further, there have been claims that teachers' pedagogical choices can strongly influence the effectiveness of iPads for engaging students in mathematics. This paper presents an investigation of the influence of teaching and learning mathematics with iPads on students' attitudes and engagement in mathematics. The participants in this study were students in a large urban primary school, implementing an iPad program for teaching and learning across the curriculum. Surveys with five-point Likert-type items were used to measure students' attitudes to mathematics. Students from Years 2 to 6 completed the survey at the beginning and end of two consecutive school years. Survey results suggested that iPad use in mathematics has the potential to impact positively on students' attitudes to mathematics. At the end of the second year of the study, semi-structured interviews were conducted with teachers and students. The interview responses confirmed that iPads had a positive influence on students' engagement and attitudes to mathematics, and that the pedagogical approaches utilised by teachers for embedding iPads in their mathematics lessons contributed positively to these outcomes." – Abstract



#### 4. Play to Learn: The Impact of Technology on Students' Math Performance

Hirata, G. (2022). *Play to Learn: The Impact of Technology on Students' Math Performance*. *Journal of Human Capital*, 16(3), 437-459.

"This paper analyzes the impact of technology use on student math learning using experimental data from Brazil. The technology is a software tool designed for elementary school students to learn and practice arithmetic. Contrary to most interventions in which computer-aided instruction extends the school day, students played the game during class time for about 2 months. First, second, and third graders who used the software increased their score on a math test by 0.56 in the short term (just after intervention) and by 0.17 in the medium term (1 year after the end of the intervention)." – Abstract

#### 5. Educational Technology: An Equity Challenge to the Common Core

Kitchen, R., & Berk, S. (2016). *Research commentary: educational technology: An equity challenge to the Common Core*. *Journal of Research in Mathematics Education*, 47(1), 3-16.

"The implementation of the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) has the potential to move forward key features of standards-based reforms in mathematics that have been promoted in the United States for more than 2 decades (e.g., National Council of Teachers of Mathematics, 1989,2000; National Science Foundation, 1996). We believe that this is an especially opportune time to purposely focus on improving the mathematics education of students who have historically been denied access to a high-quality and rigorous mathematics education in the United States, specifically low-income students and students of color (e.g., Kitchen, DePree, Celedon-Pattichis, & Brinkerhoff, 2007; Leonard & Martin, 2013). We discuss a challenge to realize standards-based reforms in mathematics in the United States: computer-based interventions in mathematics classrooms." – Abstract

#### 6. Mental Number Line Training in Children with Developmental Dyscalculia

Kucian, K., Grond, U., Rotzer, S., Henzi, B., Schonmann, C., Plangger, F., Galli, M., Margin, E, von Aster, M. (2011). *Mental number line training in children with development dyscalculia*. *NeuroImage*, 57(3), 792-795.

"Developmental dyscalculia (DD) is a specific learning disability that affects the acquisition of mathematical skills in children with normal intelligence and age-appropriate school education (prevalence 3-6%). One essential step in the development of mathematical understanding is the formation and automated access to a spatial representation of numbers. Many children with DD show a deficient development of such a mental number line. The present study aimed to develop a computer-based training program to improve the construction and access to the mental number line. Sixteen children with DD aged 8-10 years and 16 matched control children completed the 5-week computer training. All children played the game 15 min a day for 5 days a week. The efficiency of the training was evaluated by means of neuropsychological tests and functional magnetic resonance imaging (fMRI) during a number line task. In general, children with and without DD showed a benefit from the training indicated by (a) improved spatial representation of numbers and (b) the number of correctly solved arithmetical problems. Regarding group differences in brain activation, children with DD showed less activation in bilateral parietal regions, which reflects neuronal dysfunction in pivotal regions for number processing. Both groups showed reduced recruitment of relevant brain regions for number processing after the training which can be attributed to automatization of cognitive processes necessary for mathematical reasoning. Moreover, results point to a partial remediation of deficient brain activation in dyscalculics after consolidation of acquired and refined number representation. To conclude, the present study represents the first attempt to evaluate a custom-designed training program in a group of dyscalculic children and results indicate that the training leads to an improved spatial representation of the mental number line and a modulation of neural activation, which both facilitate processing of numerical tasks." - Abstract

#### 6. Analysis of Elementary School Students' Mastery in Math Instruction Based on Arithmetic Gamification

Widodo, S., & Rahayu, P. (2019). *Analysis of elementary school students' mastery in math instruction based on arithmetic gamification*. *Journal of Physics: Conference Series*, 1157(4), 6

This mixed method study looks at the impact of arithmetic gamification on facility with multiplication operations among third graders in Purwakarta, Indonesia. Results show improvement in content mastery and engagement.



# Theory of Action Statement 8

*When school and administrators have access to information about student progress in math learning and skill development, they promote and foster a culture of data-informed math instruction throughout the school/district.*

*The following studies and papers support this theory of action statement and provide guidance to our product development and product design, professional learning and partner success teams when creating, developing, and implementing our program.*

## 1. Assessing and Validating Effects of a Data-Based Decision-Making Intervention

*Keuning, T., Geel, M., Visscher, A., & Fox, J. (2019). Assessing and Validating Effects of a Data-Based Decision-Making Intervention on Student Growth for Mathematics and Spelling. Journal of Educational Measurement, 56(4), 757-792.*

“Data-based decision making (DBDM) is presumed to improve student performance in elementary schools in all subjects. The majority of studies in which DBDM effects have been evaluated have focused on mathematics. A hierarchical multiple single-subject design was used to measure effects of a 2-year training, in which entire school teams learned how to implement and sustain DBDM in 39 elementary schools. In a multilevel modeling approach, student achievement in mathematics and spelling was analyzed to broaden our understanding of the effects of DBDM interventions. Student achievement data covering the period from August 2010 to July 2014 were retrieved from schools’ student monitoring systems. Student performance on standardized tests was scored on a vertical ability scale per subject for Grades 1 to 6. To investigate intervention effects, linear mixed effect analysis was conducted. Findings revealed a positive intervention effect for both mathematics and spelling. Furthermore, low-SES students and low-SES schools benefitted most from the intervention for mathematics.” - Abstract

# Theory of Action Statement 9

*When parents have access to information about their student's progress in math, they become more in touch with their student's development in math and more confident in their ability to support their student*

*The following studies and papers support this theory of action statement and provide guidance to our product development and product design, professional learning and partner success teams when creating, developing, and implementing our program.*

## 1. Who Makes the Cut? Parental Involvement and Math Trajectories Predicting College Enrollment

*Degol, J. L., Wang, M. T., Ye, F., & Zhang, C. (2017). Who makes the cut? Parental involvement and math trajectories predicting college enrollment. Journal of Applied Developmental Psychology, 50, 60-70.*

“The present study examined the associations between parental involvement and college enrollment using a national sample of 3116 U.S. youth (52% male, 70% White). Four dimensions of parental involvement (academic values, behaviors promoting future academic success, home structure, and school involvement) were examined from 7th–12th grade. Higher initial levels of all four parenting dimensions in junior high school were associated with a greater likelihood of college enrollment. Less steep declines in academic values and behaviors promoting future academic success and increases in school involvement were also associated with an increased likelihood of college enrollment. Math achievement trajectories from 8th through 12th grade were examined as mediators of these associations. Math achievement intercepts mediated the association between the parental involvement intercepts (academic values, behaviors promoting future academic success, home structure, and school involvement) and college enrollment. No mediation was detected among math achievement linear slopes. Practical implications are discussed.” -Abstract

## 2. The Reciprocal Relationship Between Parental Involvement and Mathematics Achievement

*Hong, S., Yoo, S. K., You, S., & Wu, C. C. (2010). The reciprocal relationship between parental involvement and mathematics achievement: Autoregressive cross-lagged modeling. The Journal of Experimental Education, 78(4), 419-439.*

“This study focused on comparing the longitudinal associations between two types of parental involvement (i.e., mathematics value and academic reinforcement) and high school students' mathematics achievement, using data from the Longitudinal Study of American Youth (LSAY). Results, based on multivariate autoregressive cross-lagged modeling, indicated that parents' academic reinforcement had no effect on students' mathematics achievement and vice versa; however, a statistically significant positive reciprocal influence existed between parents' mathematics value and students' mathematics achievement throughout high school. This result not only reaffirms that parental involvement is a multidimensional construct but also implies that parental involvement has a domain-specific effect. Results from multigroup analyses revealed that students' gender did not have a differential effect on these associations.”

## 3. Multidimensionality of Parental Involvement and Children's Mathematics Achievement in Taiwan

*Kung, H. Y., & Lee, C. Y. (2016). Multidimensionality of parental involvement and children's mathematics achievement in Taiwan: Mediating effect of math self-efficacy. Learning and Individual Differences, 47, 266-273.*

The present study confirmed the multidimensional construct of Taiwanese parental involvement and examined the direct and indirect influences of parental involvement on students' mathematics achievement, that is, the mediating effects of math self-efficacy. Questionnaires were administered to 1286 seventh grade students in Central Taiwan, and structural equation modeling was utilized. The results indicated that the multidimensional model of parental involvement in mathematics contained three components: parental beliefs and expectations, managerial involvement (i.e., parental instruction), and structural involvement (i.e., resources parents provide for children). In addition, results suggested that parental involvement is indirectly associated with students' mathematics achievement through the mediating effects of math self-efficacy, either partially or completely. The conceptual framework and measures can contribute to future work on parenting, including research designed to map multidimensional constructs that describe parental involvement and studies that explore how parental involvement shapes school outcomes of early adolescents in mathematics through their math self-efficacy.

#### 4. Parental Involvement in Children’s Mathematics Learning Before and During COVID-19

Panaoura, R. (2021). *Parental involvement in children's mathematics learning before and during the period of the COVID-19*. *Social Education Research*, 65-74.

“[COVID-19] has led to sudden and unexpected circumstances in education for all the involved people (pupils, teachers, education policymakers, parents). International organizations have paid attention to their responses in crises by using alternative modes of teaching. The typical teaching methods had to be replaced by e-learning processes and all the participants needed to adjust themselves and adapt innovative methods. Most studies concentrated on teachers' and students' difficulties, barriers and new challenges. However, a different role was given to parents as well, especially in the case of primary and the first grades of secondary education, as they were asked to facilitate their children to use the e-learning processes and support them during the learning process. The present study examined the change of parental involvement during the pandemic in comparison to the previous situation in the case of mathematical subject. A questionnaire was constructed and administered to parents from Cyprus at the first days of the school year 2019-2020 in order to examine their beliefs and self-efficacy beliefs about their parental role and involvement during homework at their children's mathematical understanding and the development of their children's perseverance strategies during mathematical problem solving. The same questionnaire was administered to them at the end of the e-learning teaching processes in May 2020, after they had alternative experiences in order to identify any differences at their respective beliefs. Only the sample of parents who took part at both phases of the study was used for the comparative analyses. Results indicated that parents, especially in primary education, had the willingness to support their children's learning under the different teaching and learning processes; however, they needed training in order to develop appropriate knowledge, relevant experiences, positive beliefs and self-efficacy beliefs.” - Abstract

#### 5. Disassociating the Relation Between Parents’ Math Anxiety and Children’s Math Achievement: Long-Term Effects of a Math App Intervention

Schaeffer, M., Rozek, C., Berkowitz, T., Levine, S., & Beilock, S. (2018). *Disassociating the Relation Between Parents' Math Anxiety and Children's Math Achievement: Long-Term Effects of a Math App Intervention*. *Journal of Experimental Psychology: General*, 147(12), 1782-1790.

“Although parents’ fears and worries about math – termed math anxiety – are negatively associated with their children’s math achievement in early elementary school, access to an educational math app that 1<sup>st</sup> grade children and parents use together can ameliorate this relation. Here we show that children o higher-math-anxious parents learn less math during 1<sup>st</sup>-3<sup>rd</sup> grades, but this is not the case when families are given a math app (even after app use markedly decreases). Reducing the link between parents’ math anxiety and their positive attitudes about math for their children helped to explain the sustained benefit of the math app. These findings indicate that interventions involving parents and children together can have powerful lasting effects on children’s academic achievement and suggest that changes in parents’ expectations for their children’s potential for success in math, and the value they place on this success, play a role in these sustained efforts.” – Abstract